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Application of Rayleigh probability density function in electromagnetic wave propagation

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Abstract: In this study, signal voltage obtained at the receiver is investigated by taking the Rayleigh Probability Density Function into account. Probability of received signal and occurrence of incoming signal between two levels are also studied. Success percentage, requirement of how much the receiver is to be modified and variation of voltage or power at the output with respect to time are simulated in MATLAB for various physical environments.

Keywords: Rayleigh Probability Density Function, Multipath Fading, Fade Depth, Probability of Reception of Signal, Distribution Function

1. Introduction

Rayleigh Probability Density Function (Rayleigh PDF) is used for the cases in which there is non-line of sight (NLOS)[1] between transmitters and receivers for the communication networks and channel modeling. When there occur phase differences between multipath signals arriving the receiver, fading takes place. Rayleigh modeling is used for multipath fading in modeling of change of voltage or power that will be received. In Rayleigh modeling, in contrast to the Ricean, there is no any specified direction, that is, signals coming from any direction are assumed to have equal probability.

Moreover Rayleigh Modeling is used for noise analysis. If a signal coming to a receiver via reflection becomes so greater than the direct signal that it suppresses that, in this case this type of channel modeling is done by means of Rayleigh PDF.

2. Voltage Obtained at the Receiver

In Figures 1-3 the variation of voltage obtained at the receiver for different K values representing physical environments is shown. Here K is the ratio of the power of the electromagnetic wave received directly; to the electromagnetic wave received by reflection, diffraction and scattering.

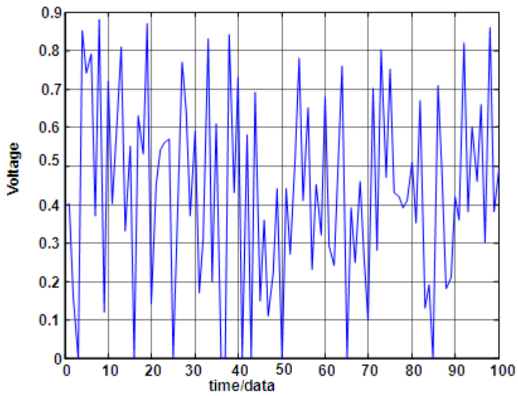

Figure 1. *The signal received by receiver for K=10 dB*

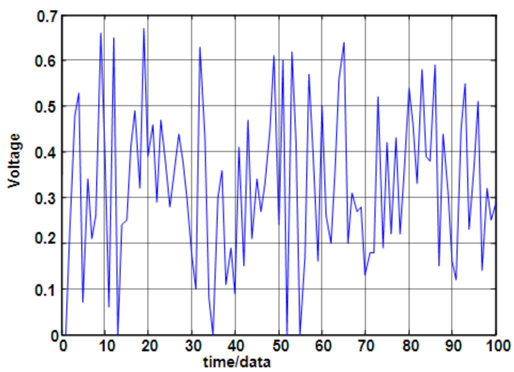

Figure 2. *The signal received by receiver for K=16 dB*

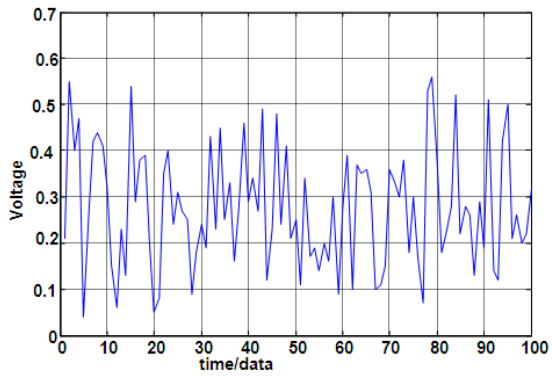

Figure 3. *The signal received by receiver for K=20 dB*

It can be seen that, as the K value increases the peak value of voltage signal obtained by the receiver decreases. It is also possible to obtain the variation of voltage at the receiver using $f_z(z)$. $f_z(z)$ is sampled in a particular time interval and this sampled values are rounded to integer numbers. If we plot these values by using a function that is distributing them randomly, the figures 1-3 are obtained [2].

3. Numerical Plot of Rayleigh Probability Density Function

It is possible to obtain Rayleigh PDF from the voltage at the receiver. Received signal is sampled in a particular time interval and this sampled values are rounded to integer numbers. when this integer numbers and their number of repetitions, probability of received signal, are plotted and put into an envelope, Rayleigh PDF is obtained. In this plot, vertical axis represents the number of repetition and horizontal axis represents the voltage level. For instance, if 0.21v is repeated 4 times among 100 samples obtained at the receiver, the probability of the occurrence of 0.21 v is 4 percent. An increase in the range of the voltage level at the receiver causes a decrease in the mean of Rayleigh PDF. This leads to a decrease in the probability values around the mean [3].

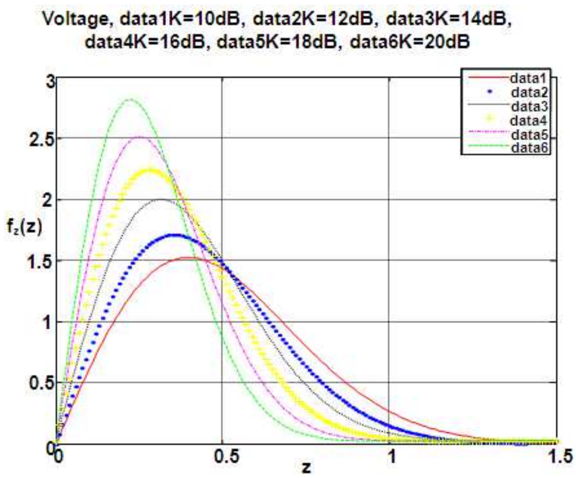

Figure 4. *The variation of $f_z(z)$ for different K values*

4. Transition From $f_z(z)$ to Time Domain

We divide $f_z(z)$ with desired steps horizontally, i.e., 0.01 increments in the example. If we distribute these 100 data values randomly to plot them, we obtain the variation of the signal with respect to time. In this function, the bigger the amplitude K gets, the more the repetition number around the mean occurs. Rayleigh PDF changes with respect to a single parameter, either standard deviation σ or K[4].

$$K = P'_{los}/P'_{multipath} \tag{1}$$

$$K = 1/2\,\sigma^2 \tag{2}$$

$$P'_{los} = 0\,Watt = 1\,dBW \tag{3}$$

K values are normalized when using in the equation (4). Rayleigh PDF is expressed as,

$$f_z(z) = 2\,zKexp\,(-(z^2K)) \tag{4}$$

When K represents the power

$$\mathrm{L = K/10} \tag{5}$$

and if it represents the voltage

$$L = K/20 \tag{6}$$

is used. For both cases

$$S = 10^L \tag{7}$$

normalized Rayleigh PDF can be written as,

$$f_z(z) = 2\,zSexp\,(-(z^2S)) \tag{8}$$

S is obtained through the equations (5), (6), (7) and used in equation (8) to obtain $f_z(z)$[5].

Mean of this function is

$$f_z(z)(mean) = M[z] = \int_0^\infty z f_z\,(z)dz, \tag{9}$$

and the variance is written as,

$$\sigma^2 = M[z^2] - M^2[z] = (0.2146/K) \tag{10}$$

and the standard deviation is

$$\sigma = 0.4632/\sqrt{\mathrm{K}} \tag{11}$$

The probability that a receiver can obtain a voltage level, is found by replacing z by the value of voltage level in $f_z(z)$.

For instance, the probability of 0.15 volt to be obtained is 2.395549 %.

In table 1, K(dB) values are converted to actual values, the mean and the standard deviation values are also shown.

***Table 1.** K values, Actual K values representing different environments, the mean and the standard deviation for power.*

K(dB)	Actual value L=K/10, S=10^L	Mean	Standard deviation (σ_z)
10	L=1; S=10^1	23.76	0.146493
11	L=1.1; S=$10^{1.1}$	21.18	0.13061
12	L=1.2; S=$10^{1.2}$	18.87	0.116396
13	L=1.3; S=$10^{1.3}$	16.82	0.103716
14	L=1.4; S=$10^{1.4}$	14.99	0.0924471
15	L=1.5; S=$10^{1,5}$	13.3	0.0823827
16	L=1.6; S=$10^{1.6}$	11.91	0.0734211
17	L=1.7; S=$10^{1.7}$	10.61	0.0654417
18	L=1.8; S=$10^{1.8}$	9.46	0.0583225
19	L=1.9; S=$10^{1.9}$	8.43	0.051972
20	L=2; S=10^2	7.51	0.0463251

In Table 2, K values, total data number and the number of data having zero probability (N_z) are given. We conclude from this table that, if K increases N_z and the probability value increases. Also this shows that low-amplitude voltages can be obtained with a higher probability than high-amplitude voltages. In this table. The data with a probability value smaller than 0.0005 are taken as zero.

***Table 2.** K values, total data number and N_z are given.*

	Total data count	N_z
K=10 dB (voltage)	10000	2096
K=12 dB (voltage)	10000	2645
K =14 dB (voltage)	10000	3184
K =16 dB (voltage)	10000	3707
K =18 dB (voltage)	10000	4207
K =20 dB (voltage)	10000	4680

***Table 3.** The relationship between K values, voltage and power means for 100 data.*

K(dB)	Environment (according to Loo's distribution model)	Voltage average	Power average
4	overall	60	47
6.3	İnfrequent light	52	36
10		42	23
11		40	21
12		38	19
13		36	17
14		34	15
15		32	14
16		30	12
17		28	11
18		27	10
18.8	frequent heavy	26	10
19		25	9
20		24	8

For voltage, when the means of the function with respect to K values are taken into consideration, it is seen that they are varying between 0.21 V and 0.6 V. For power, the variations are between 0.08 V-0.48 V. These results show that these levels can be obtained with a high probability in this voltage or power range and the remaining probabilities are close to zero. The lower-voltage levels can be obtained with a higher probability in Rayleigh model. This leads to the extension of the variation range of voltage values by lowering the threshold in the receiver. For instance, for power, the probability of 0.21 volts to be obtained at the receiver for K=10, 12, 14, 16, 18, 20 dB would be 1.11 %; 1.35 %; 1.640 %; 1.960 %; 2.312 %; 2.681 % respectively. Function behaves as a narrow band filter as the K value increases. Bandwidth is inversely proportional with K [6].

5. Obtaining Distribution Function from fz(z)

$F_z(z)$, the distribution function, is obtained by finding the area of $f_z(z)$.

$$F_z(z) = \int_0^{\infty} f_z(z)dz \tag{12}$$

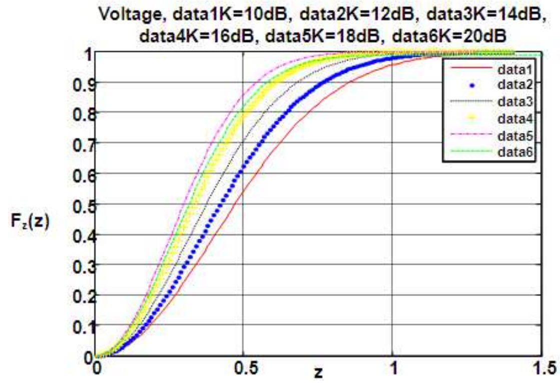

***Figure 5.** The variation of $F_z(z)$ with respect to K*

Due to the fact that the total area of $f_z(z)$ which is the sum of probabilities equals to 1. The bigger the peak value of $f_z(z)$ gets, the earlier $f_z(z)$ goes to zero, and the earlier $F_z(z)$ goes to 1. $F_z(z)$ is also quite significant as it provides us with the probability values of any desired voltage level to be less than or equal to a voltage value of z, or the probability of a voltage to occur between two voltages. In addition, this function also supplies us with the probability of the voltage or power to be obtained by the receiver or the occurrence probability of voltage or power. If we are to find the occurrence probability between any two voltages, these two points should be chosen very close, if not it would be possible to observe that the error increases linearly with the distance. In this function, the probability of voltage or power to be occur between any two points can be found with the expression given below[7]

$$F_z(z_2) - F_z(z_1) = f_z\left(\frac{z_1 + z_2}{2}\right) \tag{13}$$

For instance; the probability of a voltage between 0.23-

0.24 V to occur at the receiver is found 2.705570 % by using the equation (13). The sum of the probability values under any value can be found by using $F_z(z)$.

6. Obtaining $G_z(z)$ from $F_z(z)$

$G_z(z)$ is the failure rate and can be expressed as,

$$G_z(z) = 1 - F_z(z) \qquad (14)$$

The earlier $F_z(z)$ function goes to 1, the earlier $G_z(z)$ goes to zero. This function shows that the value of threshold at the receiver to be decreased in order to get signal at any level. The sum of the probabilities above a value can be found with this function.

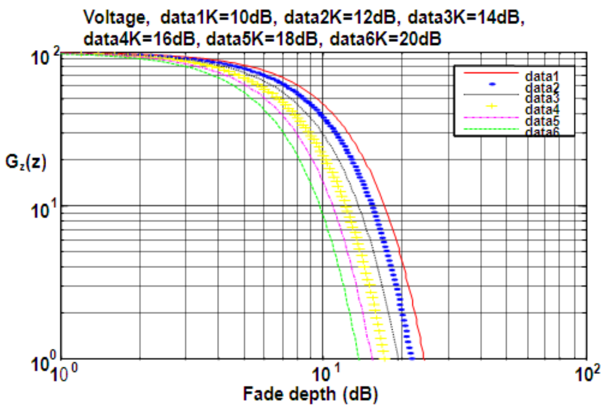

Figure 6. *The variation of $G_z(z)$ with respect to K*

In Figure 6, $G_z(z)$ is plotted for different K values. In addition, this figure shows the power to be added to the receiver in order to gain desired success rate. For instance, without any change in the system, the success rate is 0.63 % for K=10 dB. If a power of 10 dB is added to the receiver, the success rate would reach to 64 %, if 20 dB is added, the success rate would reach to 95 % [8].

7. The Variations of $f_z(z)$ and $F_z(z)$

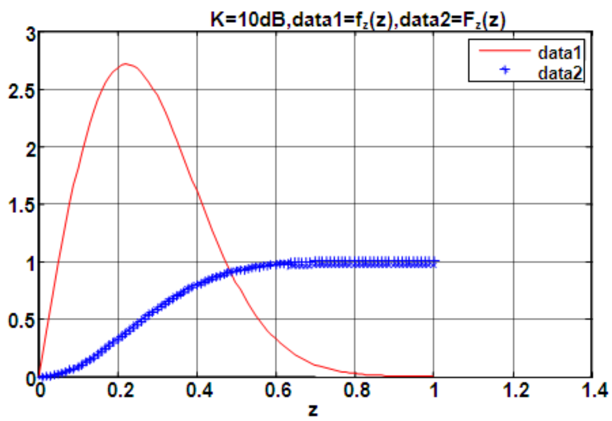

Figure 7. *The variation of $F_z(z)$ and $f_z(z)$ with respect to voltage level*

Figure 7 shows the probability of voltage values obtained at the receiver and their probability of being below any level.

8. $f_z(z)$ and $G_z(z)$ Functions

In Figure 8, the probability of voltage values obtained at the receiver and their probability of being above any level is shown [9].

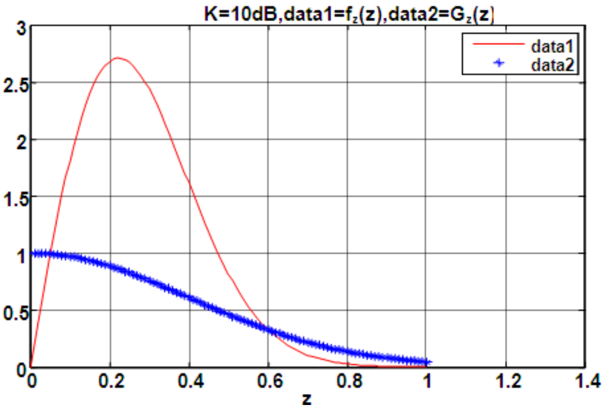

Figure 8. *The variation of $f_z(z)$ and $G_z(z)$ with respect to voltage level*

9. $F_z(z)$ and $G_z(z)$ Function

In the Figure 9, the success and failure curves of the signal obtained are shown. The curves plotted by the data 1 and data2 values, show the sum of the probabilities of being below and above a value respectively.

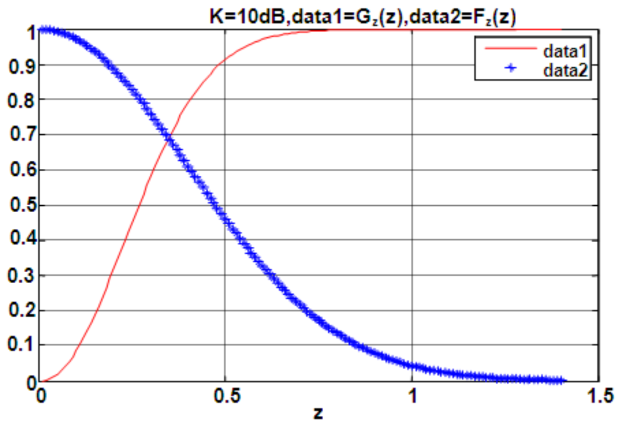

Figure 9. *The variation of $F_z(z)$ and $G_z(z)$ with respect to voltage level*

10. Conclusion

In this study, Rayleigh PDF which is obtained at the receiver according to different K values is used. The mean, and the standard deviation is calculated. Rayleigh PDF, Rayleigh distribution, failure rate and the voltage obtained at the receiver are plotted. We have interpreted the meaning of these concepts at the outdoor NLOS propagation.

It is observed that numerical plots are consistent with the theory.

References

[1] Mutlu, M. and Çavdar, İ.,Adaptation of COST-231 Walfisch-Ikegami method to Ordu city in GSM 900 frequency band. Electrical, Electronics and Computer Engineering (ELECO), National Conference, pp. 447-453, Bursa,Turkey, 2-5 Dec. 2010.

[2] Papoulis, A. Probability, Random Variables, and Stochastic Processes, McGraw-Hill, New york, 1965.

[3] Goldhirsh J. And Vogel W.J.,Handbook of Propagation Effects for Vehicular and Personal Mobile Satellite Systems, December, 1998.

[4] D. Lu and K. Yao, Improved Importance Sampling Technique for Efficient Simulation of Digital Communication Systems, IEEE Journal on Select. Areas in Commun., vol.SAC-6, no.1, pp.67–75,Jan. 1988.

[5] Mutlu, M. Minimization of the Difference between the Theoretical Mean of the Rayleigh Probability Density Function and the Mean Obtained from its Plot. Universal Journal of Electrical and Electronic Engineering pp. 23 - 29 DOI: 10.13189/ujeee.2014.020104 Vol 2 No 1, New york, Jan 2014

[6] Lee, W. C. Y. Mobile Communications Design Fundamentals, H. W. Sams and Co.,Indianapolis, Indiana, 1986.

[7] Abramowitz, M and I. Stegun,Handbook of Mathematical Functions,NBS AppliesMathematics Series, Vol. 55, U.S. Govt. Printing Office, Washington, DC.,20402.1964.

[8] Clarke, R. H, A Statistical Theory of Mobile-RadioReception, Bell System Technical Journal, Vol. 47, No. 6, July-August, pp. 329-339 .1968.

[9] Jakes, W. C.,Jr. Microwave Mobile Communications, Wiley, New york, 1974.